MATH 579: Combinatorics Exam 2

Please read the following instructions. For the following exam you may not use any papers, books, or computers. You may use a calculator. Please turn in **exactly four** problems. You must do problems 1-3, and one more chosen from 4-6. Number 7 is optional. Please write your answers on separate paper, make clear what work goes with which problem, and put your name or initials on every page. You have 50 minutes. Be sure to adequately justify all your solutions. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 20 and 40. This will then be multiplied by $\frac{5}{2}$ for your exam score.

Turn in problems 1,2,3:

1. Use difference calculus to compute $\sum_{i=1}^{100} i^4$.

2. Let $n \in \mathbb{N}$. Prove that $n\binom{2n-1}{n-1} = \sum_{k=0}^{n} k\binom{n}{k}^2$.

3. Let $n \in \mathbb{N}$. Prove that $n(n+1)2^{n-2} = \sum_{k=0}^{n} k^2\binom{n}{k}$.

Turn in exactly one more problem of your choice:

4. Let $k \in \mathbb{Z}$, $x \in \mathbb{R}$ with $x > k - 2 \geq 0$. Prove that $\binom{x}{k}\binom{x+2}{k} \leq \binom{x+1}{k}^2$.

5. Let $n, k \in \mathbb{Z}$ with $n > 1$ and $k > 1$. Prove that $k^n < \binom{nk}{n}$.

6. Compute $\sum_{k=1}^{n} \frac{H_k}{(k+1)(k+2)}$.

You may also turn in the following (optional):

7. Describe your preferences for your next group assignment. (will be kept confidential)